

Disorder-induced enhancement of the persistent current for strongly interacting electrons in one-dimensional rings

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Abstract. – We show that disorder increases the persistent current of a half-filled one-dimensional Hubbard-Anderson ring at strong interaction. This unexpected effect results from a perturbative expansion starting from the strongly interacting Mott insulator ground state. The analytical result is confirmed and extended by numerical calculations.

The interplay between disorder and electron-electron interactions is a challenging problem, at the heart of the understanding of recent important experimental findings. In particular, the measured persistent currents [1] in metallic mesoscopic rings are much larger than the theoretical prediction for noninteracting electrons [2], and the metallic phase appearing in two-dimensional electron gases at low-density [3, 4] is at odds with the scaling theory of localization [5] for noninteracting disordered systems.

Given the complexity of treating disorder and interactions on the same footing, only limiting cases or simplified models have been solved. In such cases, the emerging conclusion is that both, disorder and a repulsive interaction suppress the mobility and persistent currents. This suppression is expected to be even more important in mesoscopic systems of reduced dimensionality, where the role of correlations is enhanced by the poor screening.

These intuitive conclusions have been challenged in few-particle models, where it has been shown that a repulsive interaction, in the presence of strong disorder, may result in an enhancement of the electron mobility [6, 7]. Such delocalization effects are very dependent on the model and its particular dimensionality, and their connection with the experimentally relevant cases is still a matter of debate.

In this work we show that disorder may also have the unexpected effect of favoring the zero-temperature persistent currents, in the case of strongly interacting electrons in one-dimensional half-filled Hubbard rings. This result is established from a perturbative expansion

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starting from the strongly interacting limit, and numerical calculations based on the Density Matrix Renormalization Group algorithm [8, 9]. Within a different system, namely a two-dimensional disordered Hubbard model, numerical indications of such a behavior for the finite temperature conductivity have recently been reported [10], and related with the disorder induced breaking of the particle-hole symmetry.

A large variety of analytical and numerical tools have been developed and used for the study of interacting fermions in disordered one-dimensional systems. In particular, for spinless fermions, it has been numerically shown that repulsive interactions suppress the persistent currents, provided that the disorder is not too strong [11, 12]. Only for very strong disorder, an enhancement due to repulsive interactions occurs [13–15]. In this case, there is an important difference between the behavior of individual samples (exhibiting pronounced current peaks) and ensemble averages (showing a broad region of small enhancement for moderate values of the interaction). Once the spin is included in the description (Hubbard-Anderson model), renormalization group [16] and numerical approaches [17, 18] show that, outside half filling, repulsive interactions enhance the persistent currents in weakly disordered systems. Our work extends these results towards half filling and strong disorder, showing that the interaction induced increase of the persistent current is much more important than in the spinless case.

We write the one-dimensional Hubbard-Anderson Hamiltonian as $H = H_0 + H_1$ with

$$H_0 = U \sum_{i=1}^M n_{i,\uparrow} n_{i,\downarrow} + W \sum_{i=1}^M \sum_{\sigma} v_i n_{i,\sigma} \quad \text{and} \quad H_1 = -t \sum_{i=1}^M \sum_{\sigma} \left(c_{i,\sigma}^\dagger c_{i-1,\sigma} + c_{i-1,\sigma}^\dagger c_{i,\sigma} \right). \quad (1)$$

H_0 accounts for interaction and disorder energy, and H_1 for kinetic energy. The operator $c_{i,\sigma}$ ($c_{i,\sigma}^\dagger$) destroys (creates) a particle with spin σ on site i , $n_{i,\sigma} = c_{i,\sigma}^\dagger c_{i,\sigma}$ is the occupation number operator, and the hopping element $t = 1$ sets the energy scale. The v_i are independent random variables, uniformly distributed within $[-1/2; 1/2]$. W and U denote disorder and interaction strengths, respectively. We concentrate on chains with an even number of sites M and $N = M$ electrons (half filling). The ring topology results from the boundary condition $c_0 \equiv c_M \exp i\Phi$, where the phase $\Phi = 2\pi\phi/\Phi_0$ accounts for a magnetic flux ϕ threading the ring, and $\Phi_0 = hc/e$ is the flux quantum.

The flux dependence of the many-body ground state energy $E(\Phi)$ is periodic with period 2π . In localized systems, the persistent current at zero temperature $I = -\partial E_0/\partial\Phi \approx (\Delta E/2) \sin \Phi$ can be characterized by the phase sensitivity $\Delta E = E(0) - E(\pi)$, measuring the difference between the ground-state energies at periodic ($\Phi = 0$) and anti-periodic ($\Phi = \pi$) boundary conditions. We shall concentrate in the sequel on two related quantities: ΔE and the stiffness $D = M|\Delta E|/2$, which can be linked to the conductivity [19] and the conductance of the system [20].

At half filling, strong repulsive interactions lead to a Mott-Hubbard transition and thereby to a strong suppression of the persistent current [13]. For $U \gg t$, the kinetic energy part H_1 represents a small perturbation to the dominating term H_0 of the Hamiltonian (1), and it is possible to expand the ground-state energy in terms of t/U [15, 21–23]. Since H is symmetric with respect to spin rotation and conserves the projection S_z of the total spin S onto the z -axis, we choose to work in the subspace which is characterized by $S_z = 0$, without restricting S . Within this subspace, the N -body eigenstates $|\psi_\alpha\rangle$ of H_0 are given by products of on-site states, and completely specified by the functions $i_k^{\uparrow(\downarrow)}(\alpha)$ that select the sites which are occupied by the $N/2$ $\uparrow(\downarrow)$ -electrons ($k = 1, 2, \dots, N/2$) in the configuration α . They read

$$|\psi_\alpha\rangle = \left(\prod_{k=1}^{N/2} c_{i_k^{\uparrow(\downarrow)}(\alpha), \uparrow}^\dagger \right) \left(\prod_{k=1}^{N/2} c_{i_k^{\uparrow(\downarrow)}(\alpha), \downarrow}^\dagger \right) |0\rangle. \quad (2)$$

The corresponding many-body energies are $E_\alpha = E_\alpha^W + E_\alpha^U$, with the disorder contributions $E_\alpha^W = W \sum_k \left(v_{i_k^\downarrow(\alpha)} + v_{i_k^\uparrow(\alpha)} \right)$, and the interaction energies $E_\alpha^U = U \epsilon_\alpha$. We note ϵ_α the number of doubly occupied sites. For $U \gg W, t$ there is a gap separating the states without doubly occupied sites (subspace \mathcal{S}) from those having at least one doubly-occupied site (subspace \mathcal{D}). The \mathcal{S} -states are Mott-insulator configurations ($\epsilon_\alpha = 0$) and all have the same lowest energy $E_0^W = W \sum_i v_i$, independent of their spin structure. This degeneracy is broken by the hopping part H_1 of the Hamiltonian. Restricting ourselves to second order, we obtain an antiferromagnetic Heisenberg Hamiltonian as the effective model within the subspace \mathcal{S} [24]. Whenever neighbouring sites (i and $i-1$) have opposite spins, virtual hoppings $i \rightarrow i-1 \rightarrow i$ and $i-1 \rightarrow i \rightarrow i-1$ lead to a flux-independent coupling $-2t^2/[U(1 - (v_i - v_{i-1})^2 W^2/U^2)]$ between the two states that differ only in the orientation of the i and $i-1$ spins.

According to a theorem by Marshall, the non-degenerate ground state of the antiferromagnetic Heisenberg Hamiltonian (with energy E_0) can be written as $|\chi_0\rangle = \sum_{\beta \in \mathcal{S}} f_\beta |\psi_\beta\rangle$, where the weights f_β can be chosen real and positive with $0 < f_\beta < 1$ [24]. The effective model also contains diagonal matrix elements which are given by the sum of the above mentioned terms over all pairs of adjacent sites with opposite spin. The two configurations with alternating spins ($i_k^{\uparrow(\downarrow)} = 2k$ and $i_k^{\downarrow(\uparrow)} = 2k-1$) have the lowest diagonal elements matrix elements and therefore the largest weights f_β .

We calculate the leading contribution to the phase sensitivity ΔE , using the difference of the higher order corrections for the ground-state energies at periodic ($\Phi = 0$) and anti-periodic ($\Phi = \pi$) boundary conditions. The leading flux-dependent corrections to the ground-state energy appear in M^{th} order [23] as

$$E^{(M)} = \sum_{\gamma_1, \gamma_2, \dots, \gamma_{M-1}} \sum_{\beta, \beta'} f_\beta f_{\beta'} \frac{\langle \psi_\beta | H_1 | \psi_{\gamma_1} \rangle \langle \psi_{\gamma_1} | \dots | \psi_{\gamma_{M-1}} \rangle \langle \psi_{\gamma_{M-1}} | H_1 | \psi_{\beta'} \rangle}{(E_0 - E_{\gamma_1})(E_0 - E_{\gamma_2}) \dots (E_0 - E_{\gamma_{M-1}})}. \quad (3)$$

β and β' run over all the \mathcal{S} -states and the γ 's over the intermediate \mathcal{D} -states $|\psi_\gamma\rangle$. The numerator of the terms in (3) is composed of matrix elements $\langle \psi_{\gamma_l} | H_1 | \psi_{\gamma_{l+1}} \rangle$ of the hopping part of the Hamiltonian, thus non-zero contributions to $E^{(M)}$ arise only when all subsequent virtual states $|\psi_\gamma\rangle$ can be connected by one-particle hopping processes [23]. The resulting correction to the ground-state energy $E^{(M)}$ can be expressed using sums over sequences $\mathbf{A}^{(\beta, \beta')}$ of one-particle hopping processes, starting at $|\psi_{\beta'}\rangle$ and ending at $|\psi_\beta\rangle$.

The dependence of the corrections (3) on the boundary condition enters *via* the hopping terms between sites $1 \leftrightarrow M$ whose sign is reversed for anti-periodic boundary conditions. For periodic boundary conditions, all non-zero hopping elements are $-t$ and the numerators in (3) are given by $(-t)^n \text{sign} P_\uparrow(\mathbf{A}) \text{sign} P_\downarrow(\mathbf{A})$, with $P_{\uparrow(\downarrow)}(\mathbf{A})$ being the permutation of the positions for $\uparrow(\downarrow)$ electrons on the ring, resulting from the sequence \mathbf{A} . For anti-periodic boundary conditions, an additional sign $(-1)^{h_b}$ appears, where h_b is the number of hops across the boundary $1 \leftrightarrow M$ contained in \mathbf{A} .

Only sequences \mathbf{A} with odd h_b yield energy corrections whose sign depends on the boundary condition. They are the only contributions to the phase sensitivity ΔE . The lowest order sequences having odd h_b (and connecting one ground-state component $|\psi_\beta\rangle$ from \mathcal{S} to another one) are those which contain exactly one hop between each given pair of neighbouring sites. All hops must be in the same direction along the ring and the net effect is the transfer of one particle around the ring. This is why we work in order M , which gives the lowest order contributions with odd $h_b = 1$ [25]. Typical sequences contributing to $\Delta E^{(M)}$ are shown in Fig. 1. Diagonal contributions ($\beta = \beta'$) occur due to processes of type (a): only electrons with a given spin move while the electrons with the opposite spin orientation remain on their

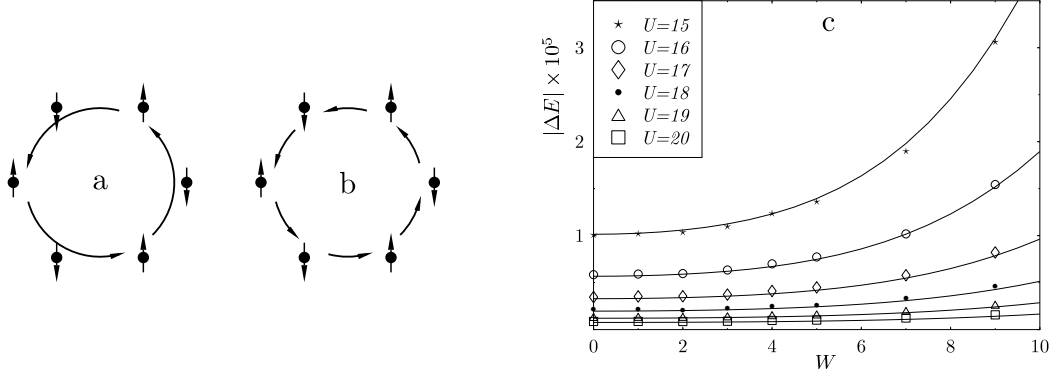


Fig. 1 – (a,b): Schematic illustration of hopping sequences of order $n = M$ contributing to the phase sensitivity (4), for the example $M = 6$. (c): The disorder dependence of $|\Delta E|$ for an individual sample with $M = N = 10$. The data (symbols) are compared to the result of our perturbation theory (lines).

initial site. Non-diagonal contributions ($\beta \neq \beta'$) can also appear. An example for which each of the electrons moves by one site is depicted in Fig. 1(b). Thus, the contributions to $\Delta E^{(M)}$ are given by those hopping sequences which contain either M forward or M backward hops. Using (3), we can express

$$\Delta E^{(M)} = \frac{-4t^M}{U^{M-1}} \sum_{\beta, \beta'} \sum_{\mathbf{A}_f^{(\beta, \beta')}} \frac{f_\beta f_{\beta'} \text{sign} P_\uparrow \left(\mathbf{A}_f^{(\beta, \beta')} \right) \text{sign} P_\downarrow \left(\mathbf{A}_f^{(\beta, \beta')} \right)}{(\epsilon_{\gamma_1} + d_{\gamma_1} W/U)(\epsilon_{\gamma_2} + d_{\gamma_2} W/U) \cdots (\epsilon_{\gamma_{M-1}} + d_{\gamma_{M-1}} W/U)} \quad (4)$$

as sums over all forward hopping sequences $\mathbf{A}_f^{(\beta, \beta')}$, with an additional factor of two accounting for the corresponding backward sequences. We have defined $E_{\gamma_l}^W - E_0^W = W d_{\gamma_l}$, and extracted the dominant parametric dependence U^{M-1} from the denominator. All contributing sequences lead to a cyclic perturbation of the $N/2$ operators corresponding to electrons with a given spin direction in (2), and since the weights f_β are all positive, the sign of the phase sensitivity at strong interaction is given by $(-1)^{N/2}$, as in the non-interacting case [23].

In order to study the effect of disorder on the phase sensitivity at strong interaction ($U \gg W$), we expand (4) in powers of W/U . Up to second order, this yields

$$\Delta E^{(M)} \approx \frac{(-1)^{N/2} 4t^M}{U^{M-1}} \sum_{\beta, \beta'} \sum_{\mathbf{A}_f^{(\beta, \beta')}} \frac{f_\beta f_{\beta'}}{\prod_l \epsilon_{\gamma_l}} \left(1 - \frac{W}{U} \sum_l \frac{d_{\gamma_l}}{\epsilon_{\gamma_l}} + \frac{W^2}{U^2} \left(\sum_l \frac{d_{\gamma_l}^2}{\epsilon_{\gamma_l}^2} + \sum_{l < m} \frac{d_{\gamma_l} d_{\gamma_m}}{\epsilon_{\gamma_l} \epsilon_{\gamma_m}} \right) \right). \quad (5)$$

Since the matrix elements of the antiferromagnetic Heisenberg Hamiltonian are functions of $(W/U)^2$, the components f_β of its ground state must be even functions of W/U . Up to second order, $f_\beta \approx f_\beta^{(0)} + f_\beta^{(2)} W^2/U^2$. The normalization condition $\sum_\beta f_\beta^2 = 1$ implies $\sum_\beta f_\beta^{(0)} f_\beta^{(2)} = 0$, and since $f_\beta^{(0)} > 0$ we must have positive and negative coefficients $f_\beta^{(2)}$. The dominating term in (5), corresponding to the clean limit $W \rightarrow 0$, exhibits an interaction-induced suppression of the persistent current $\propto U(t/U)^M$. This suppression gets exponentially more pronounced as the system size increases. As in the spinless case [15], this can be interpreted as an interaction-dependent localization length $\xi(U) = 1/\ln(U/t)$.

In contrast to the cases of lower filling or spinless fermions, where the linear correction in

W decreases the phase sensitivity [23], in the half-filled Hubbard-Anderson chain, the first-order correction in W/U vanishes exactly. We demonstrate this in the following way: a given sequence of hops can be characterized by the subsequent positions of doubly occupied sites and empty sites. Starting from this, one can construct a second sequence by exchanging the positions of the doubly occupied and empty sites. These two sequences have the same numbers of doubly occupied sites ϵ_{α_l} , and the coefficients d_{α_l} have opposite sign. Their contributions to the first-order correction in W/U cancel each other, and the sum over all sequences of the linear term in the expansion (5) vanishes. The same argument applies to higher odd orders of the expansion, all of which vanish.

Thus, the second order term of (5) determines the disorder dependence of the persistent current at $U \gg W$. The first component of its prefactor is a sum over $M-1$ positive quantities of order one, and can be estimated to $\sum_l (d_{\alpha_l}/\epsilon_{\alpha_l})^2 \lesssim M$ per sequence. The second component is a sum over terms which are also of order one, but do not have a preferential sign. In the sum over all sequences, this yields, assuming random signs, a correction which scales with the square root of the number of sequences $N_{\mathbf{A}} \gtrsim M!$, and can thus be neglected as compared to the first component which scales $\propto N_{\mathbf{A}}$. The same holds for the second order term emerging from the expansion of the f_{β} . Further, the fourth order correction (not explicitly written in Eq. 5) is dominated by a positive term yielding a prefactor of the order $M^2/2$ per sequence. This leads to the important conclusion that *disorder increases the phase sensitivity (and therewith the stiffness) and the persistent current of strongly interacting electrons*. Moreover, the relative importance of the corrections increases with the system size M .

The increase is specific to half filling. At other fillings, the symmetry between doubly occupied and empty sites disappears, and the non-vanishing first order term decreases the phase sensitivity. This insight is consistent with the conclusion of Ref. [10] that particle-hole symmetry breaking leads to a disorder-induced increase of the low-temperature conductivity in two dimensions.

In order to verify and extend the analytical result derived above, we performed extensive numerical calculations for the examples of $N = 10$ particles ($5 \uparrow; 5 \downarrow$) on $M = 10$ sites and $N = 20$ particles ($10 \uparrow; 10 \downarrow$) on $M = 20$ sites, using the Density Matrix Renormalization Group algorithm [8,9]. This algorithm allows to compute the many-body ground-state energies $E(0)$ and $E(\pi)$ for the interacting disordered system with sufficient precision to evaluate $\Delta E = E(0) - E(\pi)$ [26]. For $N = 10$ (20), we found negative (positive) ΔE for all values of U and W , consistent with the non-interacting case and the analytical result for very strong interaction $\propto (-1)^{N/2}$, in agreement with the result of Ref. [23].

The disorder dependence of $|\Delta E|$ for $N = M = 10$ particles is shown in Fig. 1 (c) for a typical individual sample (disorder realization), at several large values of the interaction. The numerical results are very well described by Eq.(5) for small ratios of W/U . In order to have an agreement over a large range of disorder (like in Fig. 1 (c)), we need to go to fourth order in W . The parametric dependence $|\Delta E| = (A_0/U^9)(1 + A_1(W/U)^2 + A_2(W/U)^4)$ (solid lines) yields the values $A_0 \approx 390000$, $A_1 \approx 2.33$, and $A_2 \approx 9.36$ (valid for all curves), consistent with the above presented estimations.

In Fig. 2, we show the interaction dependence of the stiffness D for $M = N = 10$ (a) and $M = N = 20$ (b) for typical individual samples (lines) and ensemble averages (symbols), for different disorder values W , and for the clean case $W = 0$ (solid line). The ensemble averages (of $\log D$) are performed over 100 different samples (disorder realizations). At $U = 0$, the disorder leads to Anderson localization, and D is strongly suppressed by increasing disorder (note the logarithmic scale). While the interaction always reduces the stiffness in clean rings, consistent with a Luttinger liquid calculation [27], a weak repulsive interaction $U \lesssim t$ leads to an increase of the stiffness when disorder is present. Such an increase was predicted from a

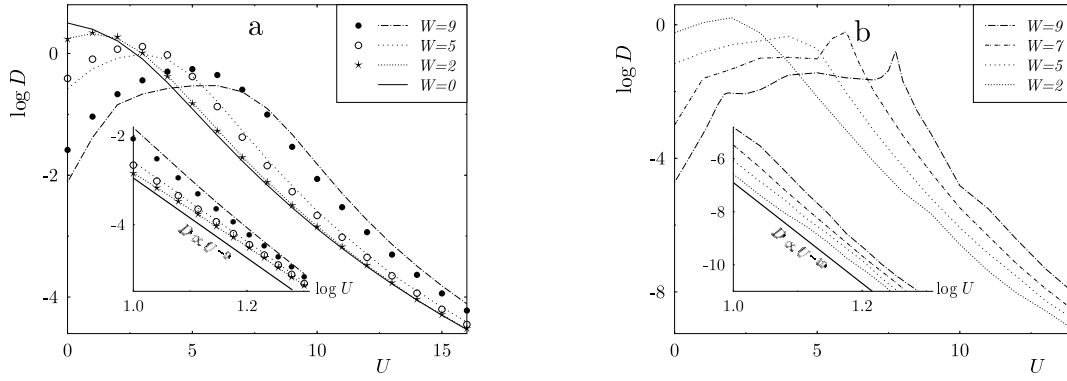


Fig. 2 – Interaction dependence of the stiffness for $M = N = 10$ (a) and $M = N = 20$ (b). Lines represent $\log D$ as a function of the interaction strength U , for different values of the disorder W , for an individual sample. Symbols in (a) correspond to the ensemble average $\langle \log D \rangle$. The statistical errors are smaller than the symbol size. In the insets, strong-interaction data are plotted on log-log scale, confirming the power law $D \sim U^{-(M-1)}$ (thick lines).

renormalization group approach [16], away from half filling. Here, at half filling, the increase becomes very pronounced as the disorder increases.

At large repulsive interaction $U \gg W$, in the Mott insulator limit, the behavior is radically different. The stiffness decreases strongly with the interaction, and, as shown in the insets of Fig. 2, the numerical results agree with the predicted power law $\propto 1/U^{M-1}$. While such an agreement is already obtained for individual samples, it persists for the average values. Moreover, and in contrast to the Anderson insulator, *the disorder increases the phase sensitivity of the Mott insulator*. This enhancement can be very strong and become larger than an order of magnitude. At intermediate interaction, between the two previous limits ($t \lesssim U \lesssim W$), the phase sensitivity exhibits a maximum which becomes broader with increasing disorder and the disorder-induced decrease of its height is much weaker than for the Anderson insulator.

Our analysis over two sizes confirms that the disorder-induced increase of the persistent currents becomes more pronounced (in relative terms) as the system size increases. In the case of $N = M = 10$, where the ensemble average can be readily performed, we observe a similar behavior for individual samples and the average. Noticeable differences only appear at very strong disorder, but the analytically predicted enhancement of the phase sensitivity is robust with respect to impurity average. For the large system size $M = N = 20$, small peaks appear in the interaction-dependence of D , in individual samples at strong disorder. This structure is less pronounced than in the case of spinless fermions in strong disorder $W = 9$ [14, 15], where sharp peaks appear at realization-dependent values of U . It is also important to notice that the data of Fig. 2 exhibit, for all finite values of the disorder, a pronounced increase of the stiffness at intermediate U (with respect to the noninteracting case). This enhancement for the average can be larger than an order of magnitude, while for spinless fermions a small maximum appears only at very strong disorder [14, 15].

In conclusion, we have investigated the effect of disorder on the persistent current of interacting electrons in one-dimensional half-filled chains, taking into account the spin degree of freedom. Within a systematic perturbative expansion of the phase sensitivity, we have unambiguously demonstrated that disorder increases the persistent current in the presence of strong interactions, in the Mott insulator limit. This is in striking contrast to the non-

interacting Anderson insulator limit, where disorder localizes the electrons and suppresses the persistent current. Such an effect can be related to the disorder-induced reduction of the energy gap between the Mott insulator ground state and excited states. Moreover, we have also shown that, for intermediate interactions between the two strongly insulating Anderson and Mott limits ($t \lesssim U \lesssim W$), the stiffness exhibits a pronounced maximum, consistently with the results away from half filling. Such an effect might be a precursor of an even stronger tendency towards interaction-induced delocalization in disordered two-dimensional systems.

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